TRINITY COLLEGE

MATHEMATICS APPLICATIONS UNITS 3 AND 4
Section One:
Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	51	35
Section Two: Calculator-assumed	12	12	100	102	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free   **35% (51 Marks)**

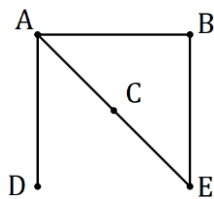This section has **seven (7)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1   **(7 marks)**

(a)   The following bipartite graph shows the subjects studied by three students. Redraw the graph to clearly show the two distinct sets of vertices and hence state which vertices represent the subjects studied.   (3 marks)


(b)   The complete bipartite graph denoted by $K_{p,q}$ has p vertices in one set and q vertices in the other set.

   (i)   Draw $K_{4,2}$ and state whether the graph is Eulerian, semi-Eulerian or neither.
                                                                                    (2 marks)

   (ii)   Draw $K_{3,2}$ and state whether the graph is a Hamiltonian cycle, semi-Hamiltonian or neither.
                                                                                    (2 marks)

Question 2   **(7 marks)**

The graph below shows a time series plot, where N is the number of breakdowns of a printing machine that are recorded during month t.
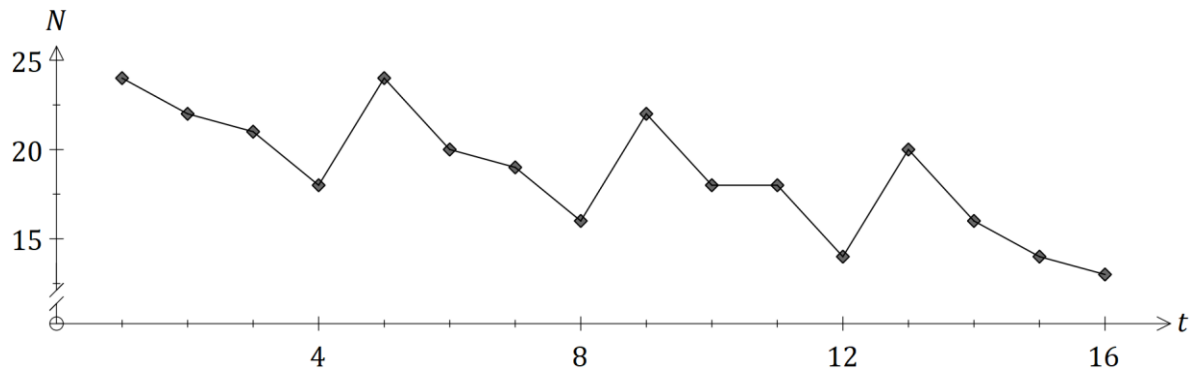
(a)   Describe the seasonality and trend of the time series.   (2 marks)

(b)   Some of the data is given in the table below.

t	3	4	5	6	7	8
N	21	18	24	20	19	16

    (i)   Calculate the 4-point centred moving average for $t = 6$.   (2 marks)

    (ii)   Explain the purpose of calculating simple moving averages for a time series.
                                                         (2 marks)

   (iii)   Explain the purpose of centring a 4-point moving average.   (1 mark)

Question 3   **(9 marks)**

(a)   The first two terms of an arithmetic sequence are displayed in the table below.

n	1	2	3	4
T_n	275	257		

    (i)   Add the next two terms to the table.   (2 marks)

    (ii)   Deduce a rule for the nth term of this sequence.   (1 mark)

(b)   The first two terms of a geometric sequence are displayed in graphical form below. Plot the likely position of the next three terms of the sequence on the graph.   (2 marks)
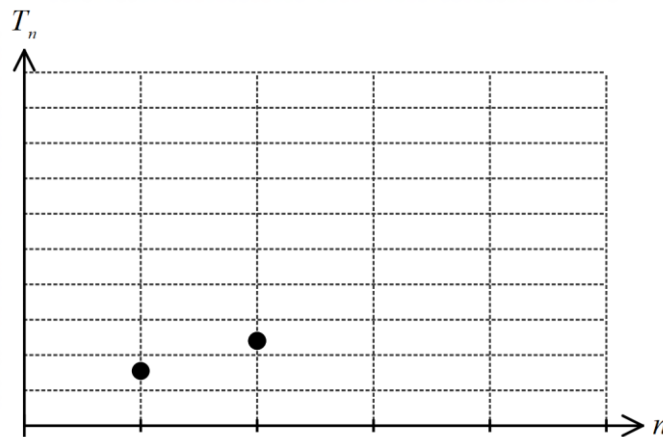
(c)   A first-order linear recurrence relation is defined by $T_{n+1} = 0.5T_n + 22$, $T_1 = 14$.

    (i)   Determine the second and third terms of the sequence.   (2 marks)

    (ii)   In the long-term, the terms of the sequence become very close to k. Determine, with justification, the value of k.   (2 marks)

See next page

Question 4 **(8 marks)**

(a)   Comment, with reasons, on the claim that the graph shown below is simple, planar and
satisfies Euler's formula. (3 marks)

(b)   A graph has 6 vertices and 8 edges. Determine the sum of the degrees of the vertices.
(1 mark)

(c)   Draw a semi-Eulerian graph that has 4 vertices and is a tree. (2 marks)

(d)   Draw a connected planar graph that has 4 vertices, 4 faces and 1 bridge. (2 marks)

See next page

Question 5 (8 marks)

The vertices in the graph below represent city landmarks and the weights on the edges are the times, in minutes, to travel between adjacent landmarks.
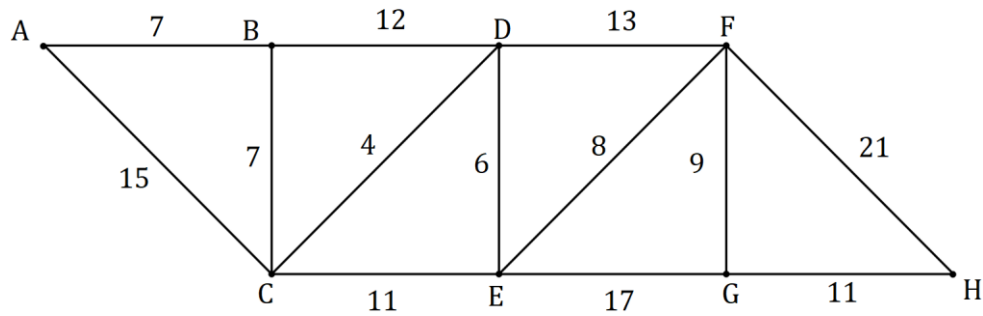
(a)   Determine the shortest path and shortest time to travel from A to H. (3 marks)

(b)   The travel times from A to B and from F to H both increase by 3 minutes. Explain how these changes affect your answer to (a). (2 marks)

(c)   Construct the adjacency matrix M for the subgraph shown below, using column and row headings in the order A, B, C, D. (2 marks)
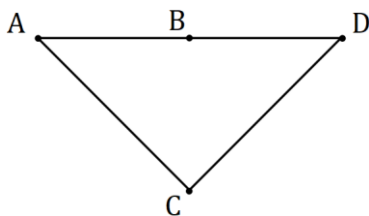
(d)   If matrix $N = M^2$, determine the value of $N_{1,1}$. (1 mark)

Question 6 **(6 marks)**

A connected graph has 6 vertices and 9 edges. The vertices represent towns and the edges represent roads between the towns. The lengths of the edges, in kilometres, are:

<div align="center">

5   6   6   7   7   7   10   10   11

</div>

(a) Determine the minimum possible length of a Hamiltonian cycle for such a graph.

(2 marks)

(b) Draw a possible graph, given that that it is also simple and has a minimum spanning tree of length 35 km. Clearly label the edge lengths and highlight the minimum spanning tree.

(4 marks)

Question 7 **(6 marks)**

Three trucks, selected from a choice of four, are to be used to carry sand from a quarry to three building sites. The table below shows the weight of sand that each truck can carry to each site per day.

	Truck P	Truck Q	Truck R	Truck S
Site 1	66	66	67	65
Site 2	61	60	63	61
Site 3	68	69	64	65

Use the Hungarian algorithm to show that the maximum amount of sand that can be transported to the three sites is 198 tonnes per day and state the required allocation of trucks to achieve this maximum.

End of questions

Supplementary page

Question number: _____

Supplementary page

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